Parameter Insensitive Control Utilizing Eigenspace Methods

Robert E. Fennell*
Clemson University, Clemson, South Carolina 26934
William M. Adams Jr.†
NASA Langley Research Center, Hampton, Virginia 23665
and
David M. Christhilf‡
Lockheed Engineering and Sciences Company, Hampton, Virginia 23666

An eigenspace assignment approach to the design of parameter insensitive control laws for linear multivariable systems is presented. The control design scheme utilizes constrained optimization techniques to exploit the flexibility in eigenvector assignments to reduce control system sensitivity to changes in system parameters while maintaining performance and robustness requirements; it thus provides a systematic approach for choosing values for eigensystem design variables. In a design example, the methods are applied to the problem of symmetric flutter suppression in an aeroelastic vehicle. In this example, the flutter mode is sensitive to changes in dynamic pressure and eigenspace methods are used to reduce the parametric sensitivity of a stabilizing minimum energy/linear quadratic regulator controller and associated observer. Numerical results indicate that the methods provide feedback control laws that make the stability of the nominal closed-loop system less sensitive to changes in dynamic pressure while maintaining acceptable control power constraints.

I. Introduction

IN this paper, a systematic procedure is presented to determine eigenspace variables so that a closed-loop system is insensitive to structured perturbations. It is assumed that an operating point model with full state feedback or dynamic compensation exhibits undesirable eigenvalue sensitivity to plant parameter variation. The design objective is to obtain a modified compensator via eigenstructure assignment methods so that the closed-loop system is less sensitive to parameter variation while performance requirements and robustness specifications are maintained. Requirements are expressed as inequality constraints that must be satisfied by a constrained optimization procedure.

The ability to shape the fundamental modes of a system by modifying selected eigenvalues and eigenvectors underlies the appeal of eigenspace methods for control design. Stability and transmission considerations motivate the location of desired eigenvalues, whereas the ability to shape the system response,[1-3] to enhance system performance,[4,5] reduce system sensitivity, and provide robust control laws[6-13] motivate the selection of desired eigenvectors.

Optimization techniques provide an effective method to determine eigenstructure design freedoms to attain desired control objectives. Two important considerations are the formulation of a cost functional and constraints to meet the design objectives and the selection of an explicit parameterization of eigenstructure design variables upon which to base a design. Recent research by Rew and Junkins[4] focuses on multicriterion optimization methods for consideration of multiple control and robustness requirements along with an eigenstructure parameterization scheme based on Sylvester's equation. These methods have been refined and applied in Ref. 5, where a least squares criterion is used to achieve well-conditioned closed-loop eigenvectors, a robustness requirement, and state and control requirements. Sequential linear programming and continuation methods are used to determine minimum eigenvalue sensitivity designs in Ref. 9. In the optimization procedure presented in this paper, multiple control objectives are expressed via penalties on control, response, and robustness variable constraint violations. Performance evaluation of the closed-loop system with either full state feedback or dynamic compensation can be incorporated in a design.

Eigenvalue sensitivity to structured perturbations can be expressed explicitly in terms of closed-loop eigenvectors, whereas sensitivity to unstructured perturbations is measured by the condition number of the closed-loop modal matrix or the orthogonality of the closed-loop eigenvectors. Consequently, the freedom available in assigning eigenvectors is directly related to sensitivity reduction[12]; see Ref. 11 for an overview of four general approaches to sensitivity reduction subject to eigenvalue placement constraints. Sensitivity reduction with respect to structured perturbations is the primary concern in this paper; stability margins as determined by the singular values of return difference matrices are used to provide a measure of robustness with respect to unstructured perturbations. Parameterizations of eigenvector design freedoms are determined by obtaining a basis for the attainable eigenvectors associated with a given eigenvalue. References 1, 2, 7, 10, 12, and 14 provide methods to obtain such parameterizations. The approach advocated here involves a precise description of attainable eigenvectors and results in the display of the design freedom in terms of a minimum number of independent parameters.

The paper is organized as follows. In Sec. II, a brief overview of eigenstructure assignment methods for feedback and observer gain matrices is presented along with an optimization procedure for parameter insensitive control design. In Sec. III, a three-input/three-output symmetric flutter suppression control problem for an aeroelastic vehicle is formulated. In this example, the flutter mode is sensitive to changes in dynamic pressure. In Sec. IV, results are presented that show an application of the parameter insensitive control design procedure to enhance the performance of a linear

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*Professor, Department of Mathematical Sciences. Member AIAA.
†Senior Research Engineer, MS 489. Associate Fellow AIAA.
‡Senior Associate Engineer, Langley Program Office, 144 Research Drive. Member AIAA.

quadratic regulator/loop transfer recovery (LQR/LTR) designed compensator for the symmetric flutter problem. Results indicate that the methods provide implementable, dynamic compensation that makes the stability of the nominal closed-loop system relatively insensitive to change in dynamic pressure while root-mean-square (rms) constraints on control deflections, rates and loads along with stability robustness constraints are maintained.

II. Parameter Insensitive Control System Design

In this section, a procedure based on eigenstructure assignment methods for the design of parameter insensitive control laws is presented. A brief overview of eigenstructure assignment methods useful in the design of full state feedback control laws and state observers is included.

Eigenstructure Assignment in Regulator Designs

Consider the linear system

$$\frac{\mathrm{d}x}{\mathrm{d}t} = Ax + Bu + Bu_0 + \Gamma\eta \tag{1}$$

where A, B, and Γ denote matrices of appropriate dimensions; x, u, η denote state, control, and disturbance variables, respectively; and u_0 represents external control inputs. With full state feedback, i.e., $u = -Kx$, the eigenvalues of $A - BK$ can be arbitrarily assigned through a proper choice of the gain matrix K if and only if Eq. (1) is completely controllable.

Numerical results presented in this paper are based on the following characterization of feedback gain matrices yielding a desired eigenstructure.[7] Let $\Lambda = \mathrm{diag}\{\lambda_i: i = 1, \ldots, n\}$ be a diagonal matrix whose diagonal elements form a distinct set of real and self conjugate complex eigenvalues and must include all uncontrollable eigenvalues of the system. There exists a feedback gain matrix K such that $(A - BK)v_i = \lambda_i v_i$ for $i = 1, \ldots, n$ if and only if

$$K = Z^{-1}U_0'(A - X\Lambda X^{-1}) \tag{2}$$

and $U_1'(\lambda_i I - A)v_i = 0$ where $B = [U_0\ U_1][Z'\ 0]$ with $[U_0\ U_1]$ an orthogonal matrix, Z a square matrix of dimension equaling the rank of B, and the modal matrix X is given by $X = [v_1, v_2, \ldots, v_n]$. If W denotes a matrix whose columns form a basis for the null space of $U_1'(\lambda_i I - A)$, then v is an attainable eigenvector of $A - BK$ corresponding to λ_i provided

$$v = Wc \tag{3}$$

where c is the vector whose components are the coordinates of v with respect to the columns of W. Equation (3) provides a parameterization/coordinization of the attainable eigenvectors associated with a desired eigenvalue. As established in Ref. 7, the dimension of $\ker U_1'(\lambda_i I - A) = m + l_i$ where $m = \mathrm{rank}(B)$ and $l_i = \dim \ker[B, \lambda_i I - A]'$. The singular value decomposition provides an efficient numerical method to compute an orthonormal basis for $\ker U_1'(\lambda_i I - A)$.

In the design example to follow, the coordinates of an attainable eigenvector become design parameters and are chosen such that the resulting feedback control system has reduced sensitivity to changes in a model parameter. The approach requires that derivatives of eigenvalues with respect to system parameters be computed. If λ is a distinct eigenvalue of a matrix $Q(q)$, depending on a parameter q, and u', v are left and right eigenvectors corresponding to λ, respectively, then[7]

$$\frac{\mathrm{d}\lambda}{\mathrm{d}q} = \frac{u'\ \mathrm{d}Q/\mathrm{d}q v}{u'v} \tag{4}$$

The dependence of this sensitivity on the eigenvectors u' and v indicates a potential reduction in sensitivity by a suitable choice of eigenvectors.

Eigenstructure Assignment in Observer Designs

In a recent paper, Kazerooni and Houpt[15] presented a procedure for loop transfer recovery based on eigenstructure assignment of observers. This procedure for observer design will be used in the example to follow. These same methods were previously applied in the design of an active flutter suppression system by Liebst et al.[13] For completeness, a brief review is presented.

Suppose now only output feedback is available and an observer is to be employed to estimate the state; then, Eq. (1) is coupled with the following output, observer, and feedback equations:

$$y = Cx \tag{5}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = Az + Bu + H(y - Cz) \tag{6}$$

$$u = -Kz \tag{7}$$

Here K and H denote feedback and observer gain matrices, respectively.

Setting $e = x - z$, one obtains $\mathrm{d}e/\mathrm{d}t = (A - HC)e + Bu_0 + \Gamma\eta$. In terms of transfer functions, $T_{zu_0}(s) = T_{xu_0}(s) - T_{eu_0}(s)$. For recovery it is desirable to have $T_{zu_0} = T_{xu_0}$ or $T_{eu_0} = 0$. The latter condition can be described in terms of transmission properties of Eqs. (1) and (5). A complex number ξ is called an invariant zero[16,17] of the system [Eqs. (1) and (5)] with left zero direction $[\nu', \mu']$ provided

$$[\nu', \mu']\begin{bmatrix} \xi I - A & B \\ C & 0 \end{bmatrix} = 0$$

and $[\nu', \mu'] \neq 0$. If a gain matrix H can be chosen which assigns the eigenvalues $\{\lambda_i: i = 1, \ldots, n\}$ of $A - HC$ to invariant zeros $\{\xi_i: i = 1, \ldots, n\}$ of Eqs. (1) and (5) and left eigenvector u_i' to corresponding left zero direction ν_i' then, necessarily,

$$[\nu_i', \nu_i' H]\begin{bmatrix} \lambda_i I - A & B \\ C & 0 \end{bmatrix} = 0$$

Moreover, since the zero state response of $\mathrm{d}e/\mathrm{d}t = (A - HC)e + Bu_0$ is

$$e(t) = \sum_{i=1}^{n} v_i u_i' B \exp(\lambda_i t) u_0(t)$$

where u_i', v_i denote left and right eigenvectors of $A - HC$ corresponding to λ_i, it follows from $u_i' B = 0$ that $e(t) = 0$, i.e., $T_{eu_0} = 0$. Thus, for recovery, it is desirable to place the eigenvalues of $A - HC$ at the invariant zeros of Eqs. (1) and (5) and the left eigenvectors at corresponding left zero directions. Once desired eigenvalues and attainable eigenvectors have been specified, the observer gain matrix can be computed using eigenstructure assignment methods. In the design example to follow, consideration of model uncertainty motivates a slight modification to this recovery procedure. A more thorough description of the recovery properties of this approach appears in Refs. 13 and 15.

Parameter Insensitive Control Design

Here, the objective is to determine the design parameters that minimize the sensitivity of certain performance variables and maintain other performance variables within prescribed bounds. By introducing an appropriate penalty function, this problem can be formulated as an unconstrained optimization problem.

In this paper, the design variables are those eigenvalues and eigenvectors of the system matrices $A(q_0) - B(q_0)K$ and $A(q_0) - HC(q_0)$, which may be modified in order to reduce sensitivity. The design parameters are the real and imaginary parts of the designated eigenvalues and the coordinates of the associated eigenvectors. In the design process employed herein, a basis for the allowable eigenvectors must be computed for each eigenvector to be modified, see Eq. (3). The coordinates of an eigenvector with respect to this basis become the design parameters that relate to eigenvector selection. If a desired eigenvalue is to be left unaltered throughout a design, that is, eigensystem design freedoms are not fully utilized, then this basis need only be computed once.

Let α denote the vector of design parameters. In the design procedure, certain performance variables, those which measure sensitivity to plant variations, are to be minimized. Let $S(\alpha)$ denote a vector of such variables. In the design example presented in Sec. IV, $S(\alpha)$ will denote eigenvalue sensitivity to plant parameter variation. Let $P(\alpha)$ denote a vector of performance variables to be kept within prescribed bounds. The prescribed lower and upper bounds for the ith component of $P(\alpha)$ are denoted by LB_i and UB_i, i.e., it is desired that $LB_i \leq P(\alpha)_i \leq UB_i$. Let $PV(\alpha)$ denote the vector of constraint violations defined by $PV(\alpha)_i = \max\{0, P(\alpha)_i - UB_i, LB_i - P(\alpha)_i\}$.

The design procedure is to choose α to minimize the performance function

$$J = ½S(\alpha)'Q_1S(\alpha) + ½PV(\alpha)'Q_2PV(\alpha) \tag{8}$$

where Q_1 and Q_2 denote positive definite weight matrices. Minimization of $½S(\alpha)'Q_1S(\alpha)$ tends to reduce sensitivity, whereas the term $½PV(\alpha)'Q_2PV(\alpha)$ represents a penalty on performance constraint violations. Based on experience, a designer may choose to vary the weighting matrices Q_1 and Q_2 in order to obtain an acceptable design.

III. Model and Control Problem Description

This work was motivated by the desire to attain symmetric flutter suppression and gust load alleviation of an aeroelastic vehicle. The planform of the vehicle wing with three control surfaces and three vertical accelerometers is depicted in Fig. 1.

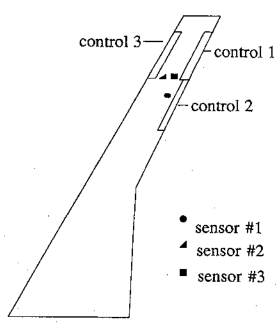

Fig. 1 Control surface and sensor locations.

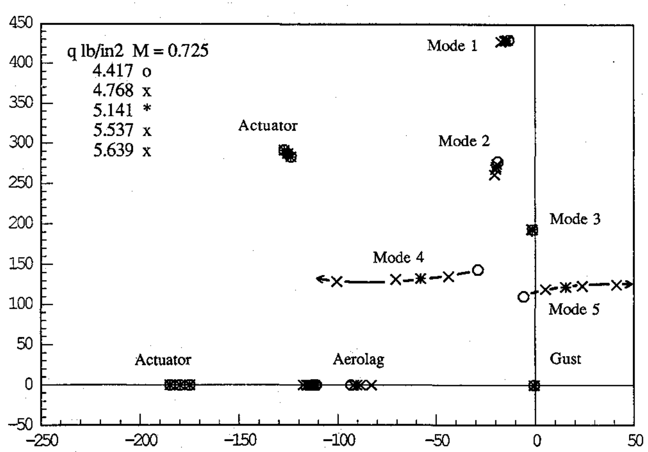

Fig. 2 Loci of open-loop poles with dynamic pressure.

State space equations for control design and evaluation were supplied by NASA.[18] These equations are of the form

$$\frac{dx}{dt} = Ax + B(u + u_0) + \Gamma\eta$$

$$y = \begin{bmatrix} C \\ C_P \end{bmatrix} x$$

where A is $n \times n$, B is $n \times r$, Γ is $n \times 1$, C is $r \times n$, and C_P is $9 \times n$ with $n = 26$ and $r = 3$. The states x are associated with elastic modes, i.e., generalized positions, rates, and unsteady aerodynamic forces, 15 states; control surface positions, rates, and accelerations, 9 states; and gusts, 2 states. The variables u and η denote control inputs and white noise into the Dryden atmospheric turbulence model, respectively; the observations Cx are associated with vertical accelerometer measurements, whereas the performance variables C_Px include control surface deflections and rates as well as incremental loads due to gust excitation. The load increments at the wing root are wing bending moment, shear, and torque. The B matrix is independent of dynamic pressure. For a Mach number of 0.725, root loci for the open-loop system, obtained by varying altitude, are depicted in Fig. 2. Flutter onset occurs when the damping in an elastic mode becomes zero. The highest dynamic pressure corresponds to a value 44% above that at flutter. This increase corresponds to what would be required if active controls were to provide the full 20% margin above the design dive speed for a transport aircraft. The point chosen for control design is $q_0 = 5.141$ lb/in.^{2}, which is approximately 11.5% above the uncontrolled flutter dynamic pressure. Other points serve as evaluation points. At the stable point $q = 4.417$ lb/in.^{2}, the open-loop incremental loads at the wing root are bending moment 25,194 in.-lb, shear 466.8 lb, and torque 1502 in.-lb. These loads serve as reference values for the design example to follow.

The control objective is to design feedback and observer gain matrices that stabilize the system over a range of values of dynamic pressure and which keep control surface deflections and rates within prescribed bounds. The maximum deflections and rates are 15 and 740 deg/s, respectively. The control law is to be designed so that saturation of controls does not occur for an input gust spectrum having an rms intensity of 12 ft/s. Typical upper and lower bounds on rms levels used in the design example to follow are listed in Table 1. To lessen the likelihood of actuator rate saturation, one would probably restrict the rms deflections and rates even further for an actual implementation. As stability robustness constraints, lower bounds are placed on the minimum singular value of the return difference matrices for the system broken at the input and output over a given frequency range.

IV. Design Example

Preliminary Design

At the design point $q_0 = 5.141$ (lb/in.^{2}), a stabilizing, minimum energy, full state feedback control law $u = -K(q_0)x$ was determined using linear quadratic regulator methodology; i.e., weighting matrices for control and state variables were chosen to be the identity and zero matrices, respectively. With such a control law, unstable open-loop poles are moved to their mirror image in the left half plane while other poles remain unaltered.

Table 1 Activator and rms performance constraints

	Control deflections, deg	Control rates, deg/s	Bending moment, in.-lb	Shear, lb	Torque, in.-lb
Lower bound	0	0	0	0	0
Upper bound	3	372	30,000	1000	3000
Nominal			25,194	466.8	1502

At the design point, the controlled system

$$\frac{\mathrm{d}x}{\mathrm{d}t} = [A(q_0) - B(q_0)K(q_0)]x + \Gamma(q_0)\eta$$

exhibits good performance as indicated by the rms values listed in column LQR of Table 2 for control surface activity and loads due to a Dryden gust spectrum having a 12-ft/s rms gust velocity. The loci of eigenvalue locations with variation in dynamic pressure for the system with full state feedback are depicted in Fig. 3. The need for further reduction in sensitivity to changes in dynamic pressure, a structured perturbation, is indicated by the instability at the evaluation point $q = 6.639$ lb/in.^{2}.

At the design point, an observer was determined through a modification of the LTR procedure discussed in Sec. II. The basic idea is to place the eigenvalues of $A - HC$ near the transmission (invariant) zeros of the plant, $C(q_0)[sI - A(q_0)]^1 B(q_0)$, and corresponding eigenvectors near the left zero directions with the modification that additional damping is added to lightly damped poles. The plant eigenvalues and invariant zeros as well as actual observer assignments are listed in Table 3. Here, eigenvalues 1–10 correspond to elastic modes, (modes 1-5), 11–15 correspond to unsteady aerodynamic states, 16–24 correspond to actuator states, and 25–26 correspond to states of a shaping filter whose output, given white noise input, has the Dryden spectrum approximation of atmospheric turbulence. The observer eigenvalue assignments listed in Table 3 were made as follows: observer poles corresponding to plant transmission zeros of the same order of magnitude as the plant poles were shifted into the left half plane to obtain a 10% damping ratio. This was done to avoid compensator pole cancellations with uncertain, high-frequency, lowly damped plant transmission zeros. Observer eigenvalues associated with zeros of small relative magnitude were simply shifted further into the left half plane and were associated somewhat arbitrarily with the nearest unassigned plant eigenvalue (corresponding left zero directions were assigned to observer eigenvectors). The remaining observer eigenvalues were assigned a large negative real part and the corresponding observer eigenvectors were chosen to be the left zero directions. In all cases the observer eigenvalues are stable. Projection methods[13] were used to obtain the observer gain matrix $H_M(q_0)$.

The system with dynamic compensator is described by Eqs. (1) and (5–7). The transfer function for the plant is $C(q_0)[sI - A(q_0)]^{-1}B(q_0)$ and for the dynamic compensator $K(s) = K[sI - A(q_0) + B(q_0)K + HC(q_0)]^1 H$. Calculations for the system with dynamic compensator show that the root loci for modes 4 and 5 are essentially the same as indicated in Fig. 3. The system with compensator has good rms and performance properties (see Table 2, column LQR/LTR).

Table 2 Root-mean-square performance at design point for LQR controller and LQR/LTR compensator

	LQR	LQR/LTR
Control deflection, deg	0.571	0.712
	0.494	0.520
	0.831	0.876
Control rates, deg/s	71.3	78.7
	57.8	59.4
	97.7	100.4
Bending moment, in.-lb	27,146	27,248
Shear, lb	505.1	506.8
Torque, in.-lb	823.4	983.9

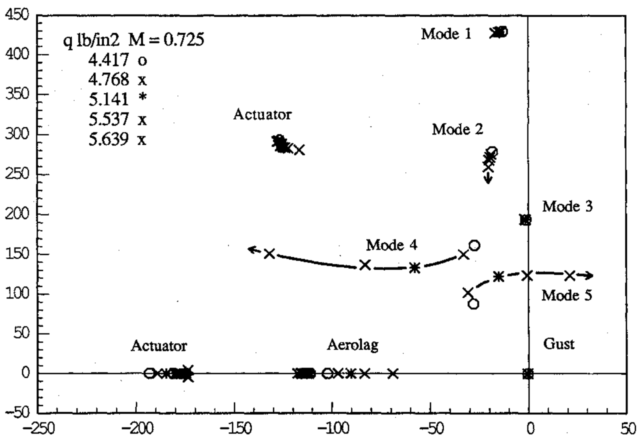

Fig. 3 Loci of poles for minimum energy feedback control.

Table 3 Eigenvalue-zero assignment for observer design

Mode	Eigenvalue	Zero	Assignment
Elastic			
1	− 14.49 + 429.2j	− 6.615 + 346.8j	− 34.68 + 346.8j
	——	——	——
2	− 19.54 + 271.8j	0.0343	− 24.0
	——	− 0.343	− 25.0
3	− 1.513 + 194.0j	− 1.044 + 198.2j	− 20.0 + 198.2j
	——	——	——
4	− 57.87 + 133.2j	0.00003 + 0.2082j	− 24.5 + 0.208j
	——	——	——
5	15.40 + 122.6j	0.000006 + 0.1040j	− 26.0 + 0.104j
	——	——	——
Aerolag	− 90.43	− 130.7	− 130.7
	− 112.8	− 112.0	− 112.0
	− 113.0	− 112.5	− 112.5
	− 113.2	− 113.0 + 0.0757j	− 113.0 + 0.066j
	− 114.5	——	——
	− 175.0	− ∞	− 874.9
Actuator	− 123.9 + 283.9j	− ∞	− 823.9 + 283.9j
	——	− ∞	——
	− 179.9	− ∞	− 879.9
	− 125.6 + 287.9j	− ∞	− 825.6 + 287.9j
	——	− ∞	——
	− 185.0	− ∞	− 885.0
	− 127.4 + 291.9j	− ∞	− 827.4 + 291.9j
	——	− ∞	——
Gust	− 0.5001	− 0.5001	− 0.5001
	− 0.4952	− 0.4952	− 0.4952

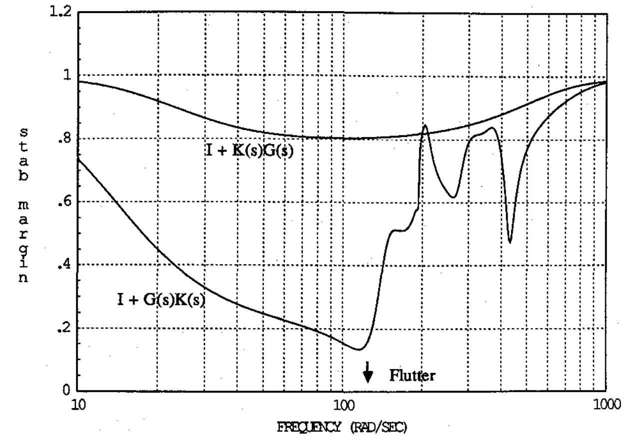

Fig. 4 Minimum singular values of $I + K(s)G(s)$ and $I + G(s)K(s)$.

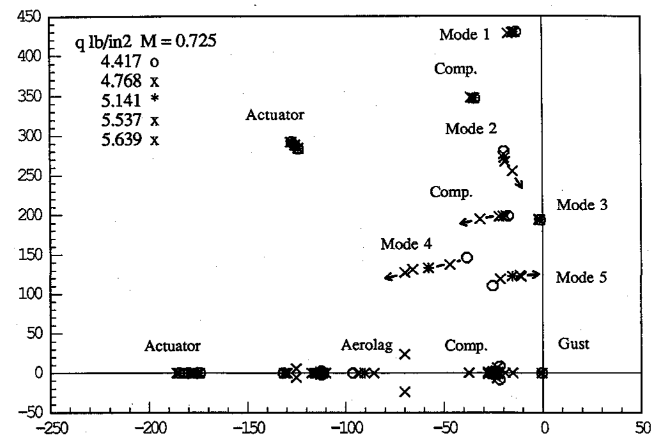

Fig. 5 Loci of poles for modified LQR/LTR compensator.

With regard to unstructured robustness considerations, one should recall[19] that the minimum energy full state feedback control law has guaranteed stability robustness properties as measured by the minimum singular value of the return difference matrix $I + K(q_0)G(s)$ only when error uncertainty is reflected at the plant input [all singular values of $I + K(q_0)G(s)$ equal 1]. LQG/LTR recovery of LQR robustness at the input is approached with the selected observer as evidenced by a minimum singular value of 0.8 for $I + K(s)G(s)$ in Fig. 4. The minimum energy controller disregards error at the output and consequently does not have guaranteed stability robustness properties when uncertainties are reflected at the plant output.[19] It is not surprising, therefore, that the LQR/LTR (input robustness recovery) exhibits poor robustness at the output (see Fig. 4).

Design Parameters and Performance Objectives

The design objective is to use the flexibility of eigenvector assignment to determine a modified full state feedback control law $K_M(q_0)$ so that the system with dynamic compensator (fixed observer) has reduced sensitivity to changes in dynamic pressure, maintains performance requirements, and satisfies robustness constraints. Many of these objectives can be achieved by simply modifying selected eigenvectors.

To illustrate the methods, a design involving only a small number of parameters is considered. Since the stabilizing, minimum energy feedback control law only alters the eigenstructure associated with the open-loop instability, a design is performed in which the closed-loop eigenvalues are left in the stable location achieved by the minimum energy controller $K(q_0)$ and an attempt is made to reduce the sensitivity of this control law to changes in dynamic pressure by modifying the eigenvectors corresponding to the unstable bending mode as well as the tightly coupled torsion mode. The design parameters $\alpha = c$ are the components of attainable eigenvectors, see Eq. (3). The modified gain matrix $K_M(q_0)$ is determined by Eq. (2) and attainable eigenvectors v corresponding to a desired eigenvalue λ must satisfy $U_1'[\lambda I - A(q_0)]v = 0$. Here, the dimension of $\ker U_1'[\lambda I - A(q_0)]$ is three for a controllable mode and four for uncontrollable modes. Normalization of attainable eigenvectors reduces the number of free parameters by one. For a controllable but fixed pair of complex eigenvalues there are four free parameters for the corresponding pair of eigenvectors. Thus, modification of bending and torsion modes provides eight free parameters.

The sensitivity term in Eq. (8) is $S(\alpha) = d\lambda/dq$, where λ denotes the unstable eigenvalue and is calculated from the formula, see Eq. (4),

$$\frac{d\lambda}{dq} = \{u'd[A(q) - B(q)K_M(q_0)]/dq|_{q=q_0}v\}/(u'v)$$

where u' and v denote left and right eigenvectors of $A(q_0) - B(q_0)K_M(q_0)$ corresponding to the unstable eigenvalue and $d[A(q) - B(q)K_M(q_0)]/dq$ is estimated by the difference quotient $\{A(q) - B(q)K_M(q_0) - [A(q_0) - B(q_0)K_M(q_0)]\}/(q - q_0)$. The sensitivity for the minimum energy controller is $d\lambda/dq = 54.27 + 7.80j$ at the design point ($q_0 = 5.141$ lb/in.^{2}).

The components of the performance vector $P(\alpha)$ are position and rate rms values for each control surface, wing bending moment, shear, and torque rms values, and, as stability robustness measures, the minimum singular value of the return difference matrices $I + K(s)G(s)$ and $I + G(s)K(s)$ over a specified frequency range. The prescribed upper and lower bounds on performance variables were determined from physical considerations (see Table 1). For the numerical results that follow, deflections for the leading-edge control surface were constrained below 3 deg in order to avoid deflections that could exceed the hinge moment capacity of the actuator. Also, a tradeoff between stability robustness to errors modeled at the input and output was sought. The tradeoff of these conflicting requirements was particularly difficult for this example, which required that the minimum energy controller closed-loop eigenstructure be maintained, whereas robustness recovery at the output corresponds, in the limit, to a controller with no constraint on control power. Consequently, to achieve a solution and to keep control effort within bounds, it was necessary to place lower bounds of 0.3 and 0.2 on the minimum singular values of $I + K(s)G(s)$ and $I + G(s)K(s)$, respectively, near the flutter frequency.

The performance index [Eq. (8)] is minimized by a search over the parameter space. The nonlinear programming algorithm used here requires no gradient computations.[20] Unequal penalty weightings in the weight matrix Q_2 in Eq. (8) were used to focus the search in appropriate directions in parameter space.

Numerical Results

Results of applying the design process wherein modifications are made to the eigenvectors associated with the bending and torsion modes are as follows. The resulting closed-loop system with optimized feedback gain matrix $K_M(q_0)$ is of the form

$$\frac{dx}{dt} = A(q)x - B(q)K_M(q_0)z + \Gamma(q)\eta$$

$$\frac{dz}{dt} = H_M(q_0)C(q)x + [A(q_0) - B(q_0)K_M(q_0) - H_M(q_0)C(q_0)]z$$

The sensitivity achieved at the design point is $d\lambda(q_0)/dq = 2.24 + 12.71j$, a substantial reduction from the LQR design. To further illustrate the sensitivity reduction of this design, the root loci for the closed-loop system with compensator are depicted in Fig. 5. Note that stability is achieved at each evaluation point, which is in marked contrast with the results of Fig. 3. Also note that there is a slight deterioration in the sensitivity of mode 2.

Values for rms control activity and loads at design and evaluation points due to a gust input with 12-ft/s noise intensity for the closed-loop system are shown in Table 4. Comparisons of design point results with those of the minimum energy controller (see Table 2) reveal rms deflections and rates increase by roughly a factor of 3 for each trailing-edge control surface. As a result of unequal penalty weightings for constraint violation in the performance index [Eq. (8)], both deflection and rate rms values for the leading-edge control surface have been reduced. The lower value for rms torque for the modified controller appears to be directly related to reduction in leading-edge control surface activity together with favorable phasing of the motion of the inboard and outboard trailing-edge control surfaces. An increase in rms outputs with increasing dynamic pressure is evident in Table 4; the sharper increase between the last two points and, more important, the lower closed-loop damping in the critical mode at the higher dynamic pressure condition (see Fig. 5). Root-mean-square control activities are well below the deflection and rate capabilities at each value of dynamic pressure.

Maximum and minimum singular values of the return difference matrices $I + K(s)G(s)$ and $I + G(s)K(s)$ for the modified full state feedback controller are shown in Fig. 6 and should be compared with Fig. 4. In order to make a small improvement in stability margins within the flutter frequency range for errors at the output, it was necessary to accept a substantial lowering of the stability margin for errors at the input over the full frequency range. It is anticipated that relaxation of the requirement that the controller maintain the eigenstructure of the minimum energy controller would allow attainment of improved robustness to errors at the output with less sacrifice in robustness to error at the input. For example, the location of the closed-loop interacting poles could be allowed to vary. Previous results[21] indicate that both the reduction in sensitivity and the robustness properties of the LQR/LTR compensator at the input can be maintained if one only considers errors at the input in the performance index [Eq. (8)]; similar results were achieved in Ref. 13.

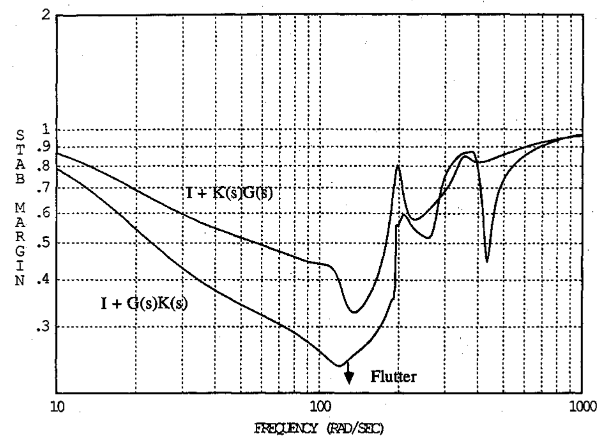

Fig. 6 Minimum singular values of $I + K(s)G(s)$ and $I + G(s)K(s)$ for the modified compensator.

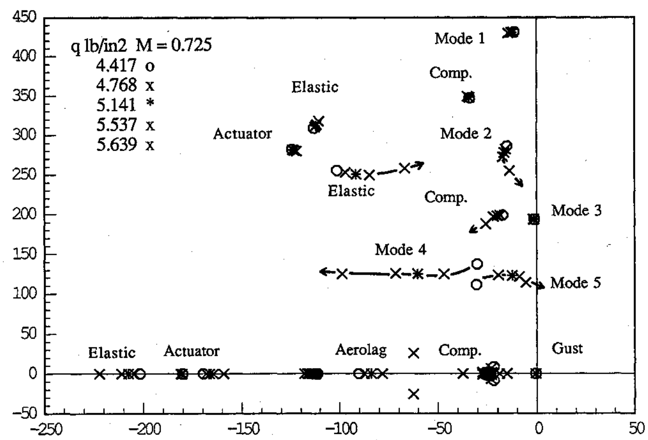

Fig. 7 Loci of poles for closed-loop system with reduced order compensator.

Table 4 Root-mean-square performance for modified LQR/LTR compensator

	Dynamic pressure, lb/in.^{2}				
rms	4.417	4.768	5.141	5.537	6.639
Control	2.57	2.76	3.01	3.32	4.33
deflections, deg	1.42	1.49	1.59	1.71	2.21
	0.502	0.547	0.608	0.690	0.950
Control rates,	180.0	204.2	235.4	274.4	383.6
deg/s	90.7	99.3	111.2	127.9	188.1
	55.9	62.3	70.7	81.5	113.5
Bending moment,					
in.-lb	25,081	26,160	27.252	28,360	31,192
Shear, lb	461.4	483.8	506.8	530.5	592.5
Torque, in.-lb	403.2	448.9	505.2	570.1	712.5

Table 5 Root-mean-square performance for evaluation model with reduced order compensator

	Dynamic pressure, lb/in.^{2}				
rms	4.417	4.768	5.141	5.537	6.639
Control	2.66	2.88	3.15	3.51	4.86
deflections, deg	1.45	1.53	1.64	1.78	2.40
	0.474	0.513	0.563	0.632	0.892
Control rates,	207.3	234.4	268.9	312.3	458.7
deg/s	108.6	119.2	133.2	152.0	225.6
	62.3	68.1	75.4	84.8	118.7
Bending moment,					
in.-lb	25,092	26,175	27.273	28,393	31,345
Shear, lb	461.8	484.4	507.6	531.7	597.2
Torque, in.-lb	412.2	459.5	519.0	591.7	840.4

In order to more fully evaluate the control design, standard model reduction techniques were used to obtain a more implementable controller. More precisely, truncation and residualization techniques were used to reduce the order of the compensator from 26th to 10th order. The root loci with dynamic pressure for the closed-loop system with reduced order compensator are depicted in Fig. 7; note that stability is maintained at each evaluation point. Root-mean-square control activity and loads for the closed-loop system with reduced order compensator are given in Table 5. Comparison with Table 4 indicates small changes in control deflections with maximum change at the largest value of dynamic pressure, increased control rates for the trailing-edge surface between 14 and 20%, and very small changes in loads except for an 18% increase in torque at the largest value of dynamic pressure.

V. Concluding Remarks

In this paper, a constrained optimization design procedure has been introduced to reduce control system sensitivities to changes in system parameters subject to other performance and robustness objectives. The procedure is based on eigenstructure assignment and utilizes a precise description of the attainable eigenvectors to display the design freedoms in terms of a minimum number of independent parameters. Eigenvector freedoms are exploited by searching within the space of attainable eigenvectors for those that satisfy the design objectives. This procedure provides a direct, albeit iterative, approach for making use of eigenvector freedoms in control law design.

The procedure was employed to design a controller for the suppression of flutter of an aeroelastic vehicle. The resulting controller made the stability of the closed-loop system less sensitive to changes in dynamic pressure than a reference minimum energy controller while maintaining control power constraints. For the example application, which sought a closed-loop system that retained the eigenstructure of a minimum control power design, small improvements in robustness to errors at the output could be attained only at the expense of a significant decrease in robustness to errors at the input.

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